Superseded

Hamilton Circuits of Convex Trivalent Polyhedra (up to 18 Vertices)

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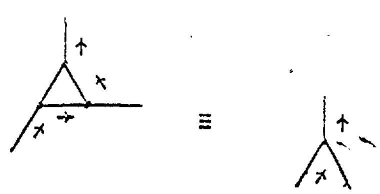
February 17, 1965

In a study of the graphs of chemical structures [1,2], it became of interest to ascertain the Hamilton circuits (a closed circuit of edges through all the vertices) of trivalent graphs, and especially of the convex polyhedra. Tait [3] had conjectured that these polyhedra always had Hamilton circuits - for brevity we will now say "had polygons". However, Tutte [4] has demonstrated a counter-example with 46 vertices. Between about 12 or 14 vertices and 46, the territory has hardly been explored.

Grace [5] has recently presented a computer tabulation of the polyhedra through 18 vertices, affording a convenient opportunity to scan them for polygons, which were found in every instance. As Grace has noted, his criterion for isomorphism: "equisurroundedness" of the sets of faces is not strictly sufficient and his list may still be incomplete. As the isomorphism of polygons is fairly readily computed, this approach may be useful in further extensions of such studies.

The work demonstrating polygonality is curtailed by the reducibility of any triangular face: A circuit through a triangle is equivalent to that through a node:



That is to say, to describe a polygonal circuit a triangular face can be shrunk down to a node. Hence, if an (n + 2)-hedron is polygonal, some (n)-hedron will be likewise. In effect, by induction, if all n-hedra are polygonal, so will be all (n + 2)-hedra with triangular faces, and we need only examine those without. Table 1 shows that only 57 forms need to be studied.

Grace displayed the polyhedra as face-incidence lists. A computer program (in Stanford University Balgol, run on an IBM 7090), translated these into vertex-incidence lists. Each vertex being identified as a face-triple, those vertices are joined which share two faces. The vertex-incidence list was then processed by a binary chained search of alternative paths; hence the search is always $<< 2^n$, in contrast to the n! scope of some permutation problems.

Table 2 displays a polygon for each of the 57 polyhedra. The other polyhedra of order ≤ 18, and some of higher order can be developed from these by expanding nodes into triangles, a process that can be iterated.

The polygons lend themselves to a compact code from which the graph of a polyhedron is quickly constructed. Draw a polygon with vertices marked 1(1)n. Each successive character of the code denotes the span of a chord drawn from the next vacant vertex. Thus the prism would be BCB. There will be n/2 characters (to be sure, the last one is redundant, being fixed by its predecessors). The letters A,B,C.... stand for spans of 1,2,3.... vertices. A and B do not appear in our list; A would connote a self-looped edge and B a triangular face.

Only one of the sometimes many forms of the polygon is shown. This is merely the first one discovered by the computer search. The examples through a_{12} are, however, in canonical form according to [2].

Table 1

COUNT OF TRIVALENT CONVEX POLYHEDRA

Vertices n	Faces f	Count total[5]	Count no triangles present
4	4	1	0
6	5	1	0
8	6	2	1
10	7	5	1
12	8	14	2
14	9	50	5
16	10	233	1~~1~~ 2
18	11	1249	35

Table 2

LISTING OF HAMILTON CIRCUITS

(Included are convex trivalent polyhedra with $n \leq 18$ vertices. Only polyhedra with no triangular face are listed. See text for code. Each character group stands for one polyhedron.)

n		
8	CECC	
10	CFDEC	
12	CGEGEC	CHFCFD
14		
16		
18		

will be recorded from attached listing

References

[1] Lederberg, J., DENDRAL-64, A System for Computer Construction, Enumeration and Notation of Organic Molecules as Tree Structures (NASA Scientific and Technical Aerospace Report, STAR No. N65-13158, 1964).

[2] Lederberg, J., Proc. Nat. Acad. Sci., U.S., 53, 134-139 (1965).

[3] Tait, P. G., Phil. Mag. (Series 5), 17, 30 (1884).

[4] Tutte, W. T., J. London Math. Soc., 21, 98 (1946).

[5] Grace, D. W., Computer Search for Non-Isomorphic Convex Polyhedra (Defence Documentation Center Technical Report, No. CS15, 1965).

to be recoded for table 2

TRIVALENT (NONTRIANGULAR) POLYHEDRA WITH 8 VERTICES

2 DGFAHCBE 2

TRIVALENT (NONTRIANGULAR) POLYHEDRA WITH 10 VERTICES

5 EIHGAJDCBF 5

TRIVALENT (NONTRIANGULAR) POLYHEDRA WITH 12 VERTICES

6 FKJIHALEDCBG 14
11 IHFLKCJBAGED

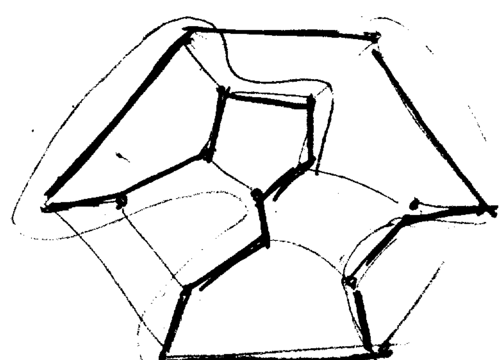
TRIVALENT (NONTRIANGULAR) POLYHEDRA WITH 14 VERTICES

6 EMJIANLKDCHGBF
7 HMGLKICAFNEDBJ 50
8 ELJIANMKDCHBGF
15 GMLKJIANFEDCBH
46 LKHGNIDCFMBAJE

TRIVALENT (NONTRIANGULAR) POLYHEDRA WITH 16 VERTICES

42 EOMKAPJNLGDICHBF
44 EOMKAJPNLFDICHBG 55 CGDIGDFD
52 HOMGLJDAPFNECKBI
54 FONIHAKEDMGPJCBL
60 EOKJAPNMLDCIHGBF
61 ENKJAPOMLDCIHBGF
62 FOLKHAPENMDCJIBG
70 LOGKJICNFEDAPHBM
88 NMIPHKJECGFOBALD
112 HONMLKJAPGFEDCBE
43 - - - - - - - -

85



TRIVALENT (NONTRIANGULAR) POLYHEDRA WITH 18 VERTICES

186	FQOIHAKEDNGRPJCMBL
195	NMGQJICLFEPHBARKDO
196	FQLJHAREODNCPKIMBG
198	NLJQIPMKECHBGARFDO
233	EQIGAMDLCPOHFRKJBN
326	POLJRIMKFDHCGQBANE
328	EQOHARKDNMGPJICLBF
329	EQKHARODNMCPJIGLBF
347	EQKJARONMDCPIHGLBF
348	EQMLARPKONHDCJIGBF
350	EQJIANMLDCPHGFRKBO
353	MLJRIQPOECNBAKHGFD
354	ELJHAQPDOCNBRKIGFM
356	MLFQHCKEPNGBAJRIDO
362	GQNMLIARFPOEDCKJBH
376	LKIHRQODCNBAPJGMFE
392	IQPMHLJEAGNFDKRCBO
393	ONMKRJQPLFDICBAHGE
401	IQPNHMJEAGROFDLCBK
418	FQNMLARKPOHEDCJIBG
419	FQMLKARPONEDCJIHBG
426	EQLKARPONMDCJIHGBF
427	EPLKARQONMDCJIHBGF
428	FQMLHAREPONDCKJIBG
429	EQJIANLKDCHGPFRMBO
477	MQGKJICPFEDOARLHBN
493	MQPHLKJDNGFEAIRCBO
505	POLGRIDKFMHCJQBANE
508	LPGKJICOFEDARQHBNM
509	POKRJIMLFECHGQBAND
625	POFRKCJMLGEIHQBAND
626	POLRJIMKFEHCGQBAND
887	IQPONMLKARHGFEDCBJ
994	ONIFHDKECRGQPBAMLJ
583	[illegible]